

Syllabus for the written examination in
Mathematics
for admission to the Ph.D. programme

Linear algebra: (50% weightage)

Vector spaces over fields, subspaces, bases and dimension. Systems of linear equations, matrices, rank, Gaussian elimination. Linear transformations, representation of linear transformations by matrices, rank-nullity theorem, duality and transpose. Determinants, Laplace expansions, cofactors, adjoint, Cramer's rule.

Eigenvalues and eigenvectors, characteristic polynomials, minimal polynomials, Cayley-Hamilton Theorem, triangulation, diagonalization, rational canonical form, Jordan canonical form.

Inner product spaces, Gram-Schmidt orthonormalization, orthogonal projections, linear functionals and adjoints. Hermitian, self-adjoint, unitary and normal operators, spectral Theorem for normal operators. Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness.

Real Analysis: (50% weightage)

Review of basic concepts of real numbers: Archimedean property, completeness. Metric spaces, compactness, connectedness (with emphasis on \mathbb{R}^n). Continuity and uniform continuity.

Monotonic functions, functions of bounded variation, absolutely continuous functions, derivatives of functions and Taylor's theorem. Riemann integral and its properties, characterization of Riemann integrable functions. Improper integrals, Gamma functions.

Sequences and series of functions, uniform convergence and its relation to continuity, differentiation and integration. Fourier series, pointwise convergence, Fejer's theorem and Weierstrass approximation theorem.