

MHRD Scheme on Global Initiative on Academic Network (GIAN)

Course Title: Positive characteristic methods in commutative algebra

Venue: Mathematics Department, IIT Bombay

Duration: 19 June-30 June 2017

Audience: Ph. D. students and post doctoral Fellows

Prerequisites: Basic Commutative algebra as covered in the Text Book: *Introduction to Commutative Algebra* by MF Atiyah and IG MacDonal*d*.

Organisers : Manoj Kummini, CMI, Chennai and J.K. Verma, IIT Bombay

Contact Details of Organisers:

J. K. Verma
Department of Mathematics,
Indian Institute of Technology Bombay,
Email : verma.jugal@gmail.com

Manoj Kummini
Chennai Mathematical Institute
Kelambakkam, Chennai, Tamil Nadu
mkummini@cmi.ac.in

Registration Fee : There is no registration fees for participants. The course is strictly for university teachers, post doctoral fellows and senior research scholars.

Local expenses for participants: The National Centre for Mathematics will provide support for local expenses of participants and speakers.

Overview: This course will focus on several interconnected themes such as invariant theory of finite and reductive groups, Anand-Dumir-Gupta conjecture about magic squares, Briancon-Skoda Theorem in complex analytic geometry and its generalisation to rational singularities and Hochster-Roberts Theorem about Cohen-Macaulay property of the rings of invariants of reductive groups in characteristic zero. These apparently disconnected topics are connected in positive characteristic if one uses tight closure of ideals and the notions of F-regular and F-rational rings.

Teaching Faculty

Name of Faculty	Title	Affiliation	Lec	Tut
H Ananthnarayan	Review of commutative algebra	IIT Bombay	5	3
R. V. Gurjar	Invariant theory of reductive groups	IIT Bombay	5	3
Anurag Singh	Positive Characteristic methods in commutative algebra	University of Utah	10	10
Manoj Kummini	Invariant theory of finite groups	CMI, Chennai	4	3
J. K. Verma	Invariant theory of finite groups	IIT Bombay	4	3

Lecture and Tutorial Schedule

Date/ Time	9.00 10.15	10.15 11.30	11.30 11.45	11.45 1.00	1.00 2.30	2.30 3.30	3.30 4.00	4.00 5.00	5.00 5.30
Mon 19	Ananth	Gurjar	Tea	Verma	Lunch	T1	Tea	T2	Snacks
Tue 20	Ananth	Gurjar		Verma		T3		T4	
Wed 21	Ananth	Gurjar		Verma		T5		T6	
Thu 22	Ananth	Kummini		Verma		T7		T8	
Fri 23	Ananth	Gurjar		Kummini		T9		T10	
Sat 24	Kummini	Gurjar		Kummini		T11		T12	
	9.30 11.00	11.00 11.30		11.30 1.00	1.00 2.30	2.30 3.30	3.30 4.00	4.00 5.00	5.00 5.30
Mon 26	Singh	Tea		Singh	Lunch	T13	Tea	T14	Snacks
Tue 27	Singh			Singh		T15		T16	
Wed 28	Singh			Singh		T17		T18	
Thu 29	Singh			Singh		T19		T20	
Fri 30	Singh			Singh		T21		T22	

Tutorial Numbers	Tutorial Instructor	Associates
T1 T5 T9	H. Ananthnarayan	Mitra Koley, JK Verma
T2 T4 T6	JK Verma	Mitra Koley, Ananthnarayan
T8 T10 T12	Manoj Kummini	Mitra Koley, Ananthnarayan
T3 T7 T11	RV Gurjar	Sudarshan Gurjar, Saurav Bhowmik
T13-T22	Anurag Singh	Tony Puthenpurakal Manoj Kummini

Lecture Schedule Course syllabus

Course 1: Review of commutative algebra by H. Ananthnarayan, IIT Bombay

- Lecture 1+2 :** Dimension Theory
Lecture 3: Cohen-Macaulay rings
Lecture 4: Gorenstein rings
Lecture 5: Completion of local rings

Course 2: Introduction to reductive groups and their invariant theory

By **R.V. Gurjar, IIT Bombay**

- Lecture 1:** Basic theory of linear algebraic groups. Lie Algebras. Examples.
Lecture 2: Reductive Groups. Several definitions using complete reducibility of finite dimensional representation, Linear reductivity, purely group theoretic definition involving the radical of the group. Examples of reductive and non-reductive group.
Lecture 3: Reynold's operator. Finite generation of the ring of invariants. Discussion of Hilbert's 14 th Problem.
Lecture 4: Properties of the quotient morphism. Luna's Etale Slice Theorem.
Lecture 5: Rationality and other properties of the singularities of the quotient.

Course 3: Invariant Theory of finite groups by J. K. Verma, IIT Bombay

- Lecture 1+2 : Noetherian and CM property of R^G , Noether's bound
Lecture 3: Examples of R^G where G is a symmetry groups of Platonic solids
Lecture 4: Molien Series and its use in computation of R^G

By **Manoj Kummini, CMI, Chennai**

- Lecture 1: Macaulay-Stanley Theorem about Gorenstein graded algebras
Lecture 2: Gorenstein property of R^G when G is a finite subgroup of $SL(n)$
Lecture 3: Shephard-Todd Theorem
Lecture 4: Some examples and results in positive characteristic

Course 4: Positive Characteristic Commutative Algebra

By **Anurag Singh, University of Utah, USA**

2 Lectures : Magic squares: Proofs of the Anand-Dumir-Gupta conjectures using positive characteristic methods. This provides an application of tight closure theory to an easily stated classical problem, introduces basic concepts such as the flatness of the Frobenius endomorphism, and illustrates the role played by F -regular rings in tight closure theory. The ADG conjectures lead to a fascinating open problem in tight closure theory, that would also be discussed here.

2 Lectures : The Hochster-Roberts theorem: Extensions of the above techniques to rings of invariants of linearly reductive groups; generic freeness, and reduction modulo p methods. These lectures would include several examples of invariant rings of classical groups such as determinantal rings and Grassmannians, and include some subtleties that arise in reduction modulo p techniques since the classical groups are linearly reductive in characteristic zero, but typically not in positive characteristic.

2 Lectures : F -rational and F -regular rings: The Hochster-Roberts theorem establishes the importance of F -regular rings and F -rational rings as an object of study in their own right. We will discuss some results and questions about these rings, as well as connections with singularities in positive characteristic including work of Smith, Hara, and Mehta-Srinivas. In these lectures, we will also discuss some important open questions in modular invariant theory that can be restated in terms of F -regular rings and splinters.

2 Lectures : The Briancon-Skoda theorem: Proofs in positive characteristic, and in characteristic zero. The theorem has a rich history, and is yet another striking application of tight closure theory. It is also an opportunity to discuss powerful techniques including the Artin-Rothaus theorem, and how it fits into the framework of general Ne'ron desingularization.

2 Lectures : Uniform bounds for symbolic powers: Theorems of Ein-Lazarsfeld-Smith and Hochster-Huneke. Bounds for symbolic powers of ideals constitute some of the most active areas of current research in commutative algebra. We will prove uniform bounds first in the case of positive characteristic, and then apply the Artin-Rothaus theorem to obtain similar results in characteristic zero. The results as well as techniques tie in very nicely with the Briancon-Skoda theorem.